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SPACE-TIME DYNAMICS IN VIDEO FEEDBACK

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Video feedback provides a readily available experimental system to study complex spatial and temporal dynamics. This article outlines the use and modeling of video feedback systems. It includes a discussion of video physics and proposes two models for video feedback dynamics based on a discrete-time iterated functional equation and on a reaction–diffusion partial differential equation. Color photographs illustrate results from actual video experiments. Digital computer simulations of the models reproduce the basic spatio-temporal dynamics found in the experiments.

1. In the beginning there was feedback

Video technology moves visual information from here to there, from camera to TV monitor. What happens, though, if a video camera looks at *its* monitor? The information no longer goes from here to there, but rather round and round the camera–monitor loop. That is video feedback. From this dynamical flow of information some truly startling and beautiful images emerge.

In a very real sense, a video feedback system is a space–time simulator. My intention here is to discuss just what is simulated and I will be implicitly arguing that video feedback is a space–time analog computer. To study the dynamics of this simulator is also to begin to understand a number of other problems in dynamical systems theory [1], iterative image processing [2], cellular automata, and biological morphogenesis, for example. Its ready availability, relative low cost, and fast space–time simulation, make video feedback an almost ideal test bed upon which to develop and extend our appreciation of spatial complexity and dynamical behavior.

Simulation machines have played a very important role in our current understanding of dynamical behavior [3]. For example, electronic analog computers in their heyday were used extensively to simulate complex behavior that could not be readily calculated by hand. They consist of function modules (integrators, adders, and multipliers) patched together to form electronic feedback networks. An analog computer is set up so that the voltages in different portions of its circuitry evolve *analogously* to real physical variables. With them one can study the response and dynamics of a system without actually building or, perhaps, destroying it. Electronic analog computers were the essential simulation machines, but they only allow for the simultaneous computation of a relatively few system variables. In contrast, video feedback processes *entire* images, and does so rapidly. This would require an analog computer of extremely large size. Video systems, however, are not as easily broken down into simple function modules. But it is clear they do simulate some sort of rich dynamical behavior. It now seems appropriate that video feedback take its proper place in the larger endeavor of understanding complex spatial and temporal dynamics.

Cellular automata are the simplest models available for this type of complexity. Their study, however, requires rapid simulation and the ability

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to alter their governing rules. Video feedback does, in fact, simulate some two-dimensional automata and rapidly, too. With a few additions to the basic system, it can easily simulate other rules. Thus video feedback has the potential to be a very fast and flexible two-dimensional automata simulator. The dynamics of cellular automata are governed by local rules, but video feedback also allows for the simulation of *nonlocal* automata. At the end, I will come back to these possibilities and describe how simulations of cellular automata, and their generalization to nonlinear lattice dynamical systems, can be implemented with video feedback.

This is largely an experimental report on the dynamics of a physical system, if you like, or a simulation machine, called video feedback. My intention is to make the reader aware of the fascinating behavior exhibited by this system. In order to present the results, however, section 2 includes the necessary background on the physics of video systems and a very straightforward description of how to start experimenting. An important theme here is that the dynamics can be described to a certain extent using dynamical systems theory. Section 3 develops those ideas and proposes both discrete and continuous models of video feedback dynamics. The experimental results, then, take the form in section 4 of an overview of a particular video feedback system's behavior and several snapshots from a video tape illustrate a little bit of the dynamical complexity.

2. Video hardware

In all feedback systems, video or other, some portion of the output signal is used as input. In the simplest video system feedback is accomplished optically by pointing the camera at the monitor, as shown in fig. 1. The camera converts the optical image on the monitor into an electronic signal that is then converted by the monitor into an image on its screen. This image is then electronically converted and again displayed on the monitor, and so on ad infinitum. The information thus flows in a single direction around the feedback loop. In fig. 1 the image information flows in a counterclockwise loop. This information is successively encoded electronically, then optically, as it circulates.

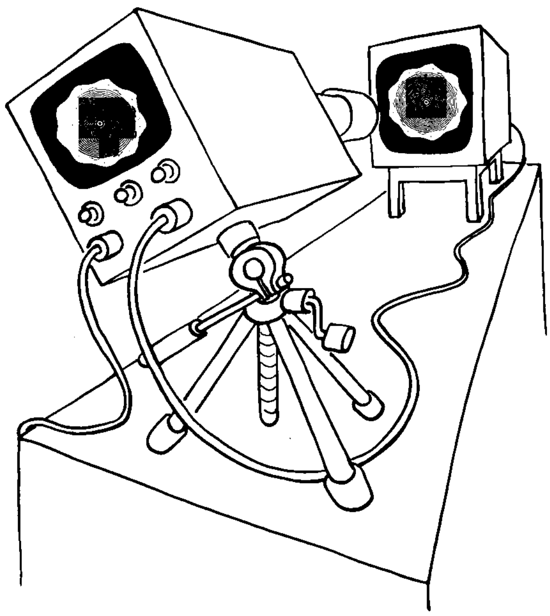

Fig. 1. Single video feedback. Information flows counterclockwise through the electronic and optical pathways.

Each portion of the loop transforms the signal according to its characteristics. The camera, for example, breaks the continuous-time optical signal into a discrete set of *rasters* thirty times a second. (See fig. 2.) Within each raster it spatially dissects the incoming picture into a number of horizontal scan lines. It then superimposes synchronizing pulses to the electronic signal representing the intensity variation along each scan line. This composite signal drives the monitor's electron beam to trace out in synchrony the raster on its phosphor screen and so the image is recontructed. The lens controls the amount of light, degree of spatial magnification, and focus, of the image presented to the camera.

Although there are many possible variations, in simple video feedback systems there are only a few easily manipulated controls. (See table I.)

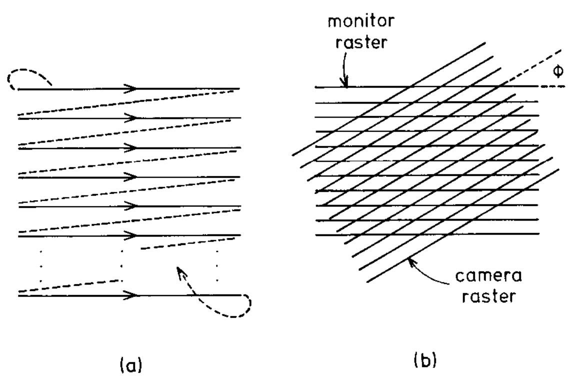

Fig. 2. Video raster with arrows indicating the direction of scanning. Solid lines correspond to when the electron beam is on; the dashed lines when the beam is off during the *retrace* time. (b) Since the raster defines the horizontal, in a feedback system the relative orientation as shown of the camera and monitor is an important control parameter.

The optical controls provide gross spatial transformations of the image seen by the camera. *Zoom*, available on most modern color cameras, conveniently allows for spatial magnification or demagnification. The same effect can be produced using a camera without a zoom lens by moving it closer to or further from the monitor. *Focus* controls image sharpness by moving the focal plane in front or behind the camera tube's image target. The total amount of light admitted to the camera is set by the *f/stop* or iris control. When pointing the camera at the monitor the relative position, or *translation*, of the raster centers and the relative angle, or *rotation*, (fig. 2b) are important controls.

Electronic transformation of the signal occurs in both the camera and the monitor. The sensitivity of the camera's tube is adjusted by a *light level* control. Some cameras also provide for *luminance inversion* that inverts the intensity of the color signals. When switched on, this allows one, for example, to view a color negative print with the camera as it would appear in a positive print. The image intensity can be adjusted again on the monitor with the *brightness*. The *contrast* controls the dynamic range of the AC portion of the intensity signal. On color monitors the amount of color in the image is set by the *color* control and the relative proportion of the primary colors (red–green–blue) is governed by the *hue*.

While the effect of each individual adjustment can be simply explained, taken together they present a formidable number of control variables

Table I
Typical control parameters on color video feedback

Name	Function
Optical	
zoom	spatial magnification
focus	image clarity
f/stop	attenuates incident light level
rotation	relative angle of monitor and camera rasters
translation	relative position of monitor and camera raster centers
Electronic	
Camera	
light level	adjust sensitivity of camera pickup tube
luminance inversion	inverts intensity signal for each color
Monitor	
brightness	varies overall intensity signal
contrast	amplifies dynamic range of intensity
color	attenuates color signals to black and white
hue	relative signal strength of colors

that can interact nontrivially. These problems will be considered in greater detail in the ensuing discussion of TV theory and possible mathematical models of feedback dynamics. This section now ends with a "cookbook" procedure for setting up a feedback system.

Although the detailed and quantitative dynamics will vary with the specific equipment used, my experience indicates that almost all servicable cameras and monitors will give some interesting behavior. This may require some patience as there are a number of controls to be properly set. But once "tuned up" a system will exhibit complex and striking imagery in a reasonably wide control range. For the movie [4] and pictures described later the camera used was a Sony Trinicon HVC-2200 and a Sony Trinitron TV/Monitor KV-1913*.

A typical start-up procedure might be as follows:

1) Connect equipment as shown in fig. 1.

2) Place camera five to six feet from monitor. The distance will depend on the monitor screen size and is not that important if the camera has a zoom lens.

3) Point camera at some object other than the monitor. Adjust camera and monitor controls to give a good image on the monitor. Vary these controls to get a feeling for their effect on the image.

4) Now turn the camera to face the monitor.

5) Again adjust the camera controls, especially the zoom and focus, noting their effect. A warning is necessary at this point: it is not a good idea to let the camera see any steady very bright image for more than 10 to 20 seconds**. Bright, dynamic moving images are generally OK.

6) Adjust camera on its tripod so that it can be tilted about its optical axis.

7) Point the camera again at the monitor, focus on the monitor front, and zoom in enough so that the "first" image of the monitor front fills 90% of the screen.

8) Slowly tilt the camera trying to maintain the camera point at the screen's center. On almost all tripods this will take some fiddling and readjustment. Try zooming in at various rotation angles between 20 and 60 degrees.

Another important element in this is the ambient light level. Some behavior is quite sensitive to, or will not appear at all if, there is any external source of light. Although, a flashlight, candle, or a quick flip of the light switch, can be good light sources to get the system oscillating again if the screen goes dark.

With this short description and a modicum of patience the experimenter has a good chance of finding a wealth of complex and fascinating spatial and temporal dynamics.

* The cost for this space–time simulator is a little over $1000, approximately a cheap home computer.

** Some new cameras incorporate "burn proof" camera tubes. They are much less susceptible than earlier cameras to the image "burn" that can permanently damage the tubes. Caution should still be exercised. Excessively bright images will shorten tube life.

3. Toward a qualitative dynamics

In the beginning, I argued that a video feedback system is a space–time simulator. But a simulator of what exactly? This section attempts to answer this question as concretely as possible at this time. A very useful tool in this is the mathematical theory of dynamical systems. It provides a consistent language for describing complex temporal behavior. Video feedback dynamics, though, is interesting not only for the time-dependent behavior but also for its complex spatial patterns. In the following section I will come back to the question of whether current dynamical systems theory is adequate for the rich spatio-temporal behavior found in video feedback.

This section introduces the qualitative language of dynamical systems [5], and then develops a set of discrete-time models for video feedback based on the physics of video systems. At the section's end I propose a continuum model akin to the reaction-diffusion equations used to model chemical dynamics and biological morphogenesis.

Dynamic, time-dependent behavior is best described in a *state space*. A particular configuration, or state, of a system corresponds to a point in this space. The system's temporal evolution then becomes the motion of an *orbit* or *trajectory* through a sequence of points in the state space. The *dynamic* is the collection of rules that specify the evolution from each point to the next in time. In many cases these rules can be simply summarized as transformations of the state space to itself by iterated mappings or by differential equations.

As will be seen shortly, video feedback is a *dissipative* dynamical system. This means that on the average "volumes" in the state space contract, or in physical terms, that energy flows through the system and is lost to microscopic degrees of freedom. This property limits the range of possible behavior. Starting from many different initial states, after a long time the system's evolution will occupy a relatively small region of the state space, this is the system's *attractor**. An attractor is *globally stable* in the sense that the system will return if perturbed off the attactor. Different initial conditions, even states very near each other, can end up on different attractors. The set of points, though, that go to a given attractor are in its *basin of attraction*. The picture for a particular dynamical system is that its state space is partitioned into one or many basins of attraction, perhaps intimately intertwined, each with its own attractor.

Very roughly there are three flavors of attractor. The simplest is the *fixed point* attractor. It is the analog to the physicist's notion of equilibrium: starting at various initial states a system asymptotically approaches the same single state. The next attractor in a hierarchy of complexity is the *limit cycle* or stable oscillation. In the state space this is a sequence of states that is visited periodically.

* Unbounded or divergent behavior can be interpreted as an attractor at infinity.

** For simplicity's sake, I have not included the predictable *torus* attractor. It is essentially the composition of periodic limit cycle attractors.

The behavior described by a fixed point or a limit cycle is predictable: knowledge of the system's state determines its future. The last type** of attractor, that is in fact a very broad and rich class, gives rise to unpredictable behavior. These are the *chaotic attractors*. While globally stable, they contain local instabilities that amplify noise, for example. They also have extremely complex orbit structure composed of unstable periodic orbits and aperiodic orbits.

An important branch of dynamical systems theory concerns how one attractor changes to another, or disappears altogether, with the variation of some control parameter. The motivation for this line of inquiry is clearly to model experimentalists's control over their apparatus. A *bifurcation* occurs when an attractor changes qualitatively with the smooth variation of control parameter. Changing controls corresponds to moving along a sequence of different dynamical systems. In the space of *all* dynamical systems, the sequences appear as *arcs* punctuated by particular control settings at which bifurcations occur. It is now known that these punctuations can be quite complex: continuous arcs themselves or even Cantor sets or fractals. The physical interpretation of these possibilities is very complex sequences of bifurcations. Thus dynamical systems theory leads us to expect not only unpredictable behavior at fixed parameters, but complex changes between those chaotic attractors.

With modifications much of this qualitative picture can be carried over to the dynamics of video feedback. It is especially useful for describing the context in which the complex behavior arises. In the following I also will point out possible inadequacies of the naive application of dynamical systems.

A single state of a video feedback system corresponds to an entire image, on the monitor's screen, say. The state is specified not by a small set of numbers, but rather a function $I(\vec{x})$; the intensity at points $\vec{x}$ on the screen. The dynamics of video feedback transforms one image into another each raster time. The domain of the intensity function $I(\vec{x})$ is the bounded plane, whereas the domain of the dynamics is the space of functions or, simply, the space of images.

This picture can be conveniently summarized by introducing some notation. The monitor screen is the bounded plane $\bar{R}^2 = [-1, 1] \times [-1, 1]$ where the coordinates of a point $\vec{x}$ take values in the range $[-1, 1]$. With this convention the center of the screen is (0, 0). For the incoherent light of video feedback, there is no phase information and so intensity is all that is significant. The appropriate mathematical description of an image's intensity distribution is the space of positive-valued functions. We will denote the space of all possible images by $\mathscr{F}$. The video feedback dynamic then is a transformation T that takes elements I in $\mathscr{F}$ to other elements: $T: \mathscr{F} \to \mathscr{F} : I \mapsto I'$.

The task of modeling video feedback is now to write down the explicit form of T using our knowledge of video system physics. To simplify matters, I will first develop models for monochrome (black and white) video feedback. With color systems the modeling is complicated by the existence of three color signals and the particular camera technology. Once the monochrome model is outlined, however, it is not difficult to make the step to color.

The construction of the monochrome model requires more detailed discussion of the electronic and optical transformations in the feedback loop. Fig. 3 presents the schematic upon which this model is based. With the physics of these transformations as discussed in the appendix, a relatively complete model can be constructed.

The appendix reviews the operation of the common *vidicon* camera tube, how it (i) stores and integrates images and (ii) introduces a diffusive coupling between picture elements. These attributes impose upper temporal and spatial frequency cutoffs, respectively. The focus turns out to be an easily manipulated control of the spatial diffusion rate. The monitor's phosphor screen also stores an image but for a time negligible compared to that

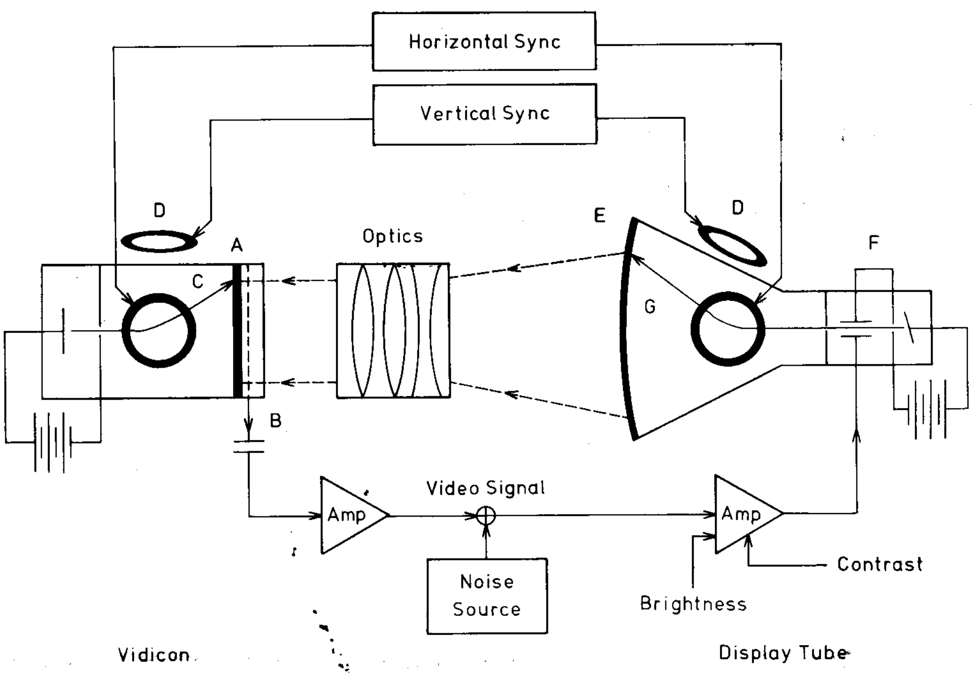

Fig. 3. Idealized monochrome video feedback. A: photoconductive image target; B: pickup for video signal; C: camera electron beam; D: scanning coils for electron beams; E: phosphor screen; F: beam intensity modulator; G: monitor electron beam.

of the vidicon. The appendix indicates various deviations from the ideal video feedback system of fig. 3.

With the physics and electronics of video systems in mind, the details of the transformation T can be elucidated for the monochrome model. The first and perhaps most significant assumption, is that T be taken as a discrete-time transformation of a spatially continuous function, the image I_n,

$$I_{n+1} = T(I_n) .$$

Employing a "bias intensity", the intensity at a point $I_n(\vec{x})$ can be scaled to take values in the range $[-1, 1]$; -1 being black and 1 white. For comparison at the end of this section, I consider how a continuous time and space model can be applied to video feedback using reaction–diffusion equations.

The new image I_{n+1} consists of two parts: the first, the "old image" stored in the photoconductor, and the second, the "incoming image" from the monitor screen. This, and the process of successive feedback of images, can be expressed as an *iterated functional equation*. The first model of the dynamic T is the following

$$I_{n+1}(\vec{x}) = LI_n(\vec{x}) + sfI_n(bR\vec{x}) , \tag{1}$$

where $\vec{x}$ is a point in $\bar{R}^2$. The first term represents the old image whose intensity at the point $\vec{x}$ has decayed by a factor of L each time step. Thus L is the intensity dissipation of the storage elements, including the monitor phosphor, but dominated by the photoconductor. The second term represents the incoming image that is possibly rotated by an angle ϕ and spatially magnified by a factor b. R is then a simple rotation,

$$R = \begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix},$$

due to the relative raster orientations; b corresponds to the zoom control. If $\vec{x}' = bR\vec{x}$ lies outside of $\bar{R}^2$ then $I_n(\vec{x}') = 0$. The parameter $f \in [0, 1]$ corresponds to the f/stop. For a system with luminance inversion black regions become white and vice versa. To take this into account the parameter s is set to -1, rather than its normal value of unity.

Spatial diffusion due to the photoconductor, but largely controlled by focus, contributes to the intensity at a point. It produces a spatial coupling to neighboring pixels that can be represented continuously by the following convolution integral:

$$\langle I_n(\vec{x}) \rangle_x = \int_{\bar{R}^2} \mathrm{d}\vec{y} I_n(\vec{y}) \exp\left(\frac{-|\vec{y} - \vec{x}|^2}{2(\sigma_f + \sigma_v)^2} \right), \tag{2}$$

assuming a Gaussian shape for the diffusion profile. The denominator in the exponential controls the width of the smoothing with σ_f representing the focus control and σ_v the intrinsic smoothing in the vidicon.

A more complete model including the major features of video feedback systems is the following:

$$I_{n+1}(\vec{x}) = LI_n(\vec{x}) + L' \langle I_n(\vec{x}) \rangle_x + sfI_n(bR\vec{x}) , \tag{3}$$

with the parameter L' setting the magnitude of the intensity signal contributed (or leaked) to that at $\vec{x}$ during one raster time.

Furthermore, the first term in eq. (3) can be modified to include the temporal storage and integration of images and their successive decay. This can be effected by a weighted sum of past images,

$$\langle I_n(\vec{x}) \rangle_\tau = \sum_{i=0}^{\infty} I_{n-i}(\vec{x}) L^i ,$$

where the decay parameter L is the same as above. This gives equations corresponding to the video feedback system as laid out in fig. 3,

$$I_{n+1}(\vec{x}) = L \langle I_n(\vec{x}) \rangle_\tau + L' \langle I_n(\vec{x}) \rangle_x + sfI_n(bR\vec{x}) . \tag{4}$$

For a color system the scalar intensity becomes a vector of red, green, and blue intensities, $\vec{I}(\vec{x}) = (R(\vec{x}), G(\vec{x}), B(\vec{x}))$. There are also couplings between the colors caused by a number of interactions and imperfections, such as

1) incorrect convergence of the monitor electron beams on the screen phosphor color dots;

2) non-ideal color filters and differential diffusion rates for the photoelectrons in the vidicon;

3) aberration in the optical system;

4) electronic cross-talk between the color signals in pickup, amplification, and reconstruction, of the image.

A model for color feedback can be developed as an extension of eq. (4) based on the evolution of a vector intensity $\vec{I}$,

$$\vec{I}_{n+1}(\vec{x}) = \underline{L}\langle \vec{I}_n(\vec{x})\rangle_\tau + \underline{L}'\langle \vec{I}_n(\vec{x})\rangle_x + sf\vec{I}_n(bR\vec{x}), \quad (5)$$

where $\underline{L}$ and $\underline{L}'$ are matrices. Their diagonal elements control the color intensity decay, while their off-diagonal elements the coupling of the color signals. In a first order approximation, this model summarizes the various couplings only linearly although it is clear that nonlinear couplings could be added.

Along the same lines a continuous-time model can be developed that for many purposes is easier to study. This also allows for the comparison of video dynamics to other work on spatial complexity in biological and chemical systems. The type of model proposed here is generally called a reaction–diffusion partial differential equation. A.M. Turing introduced this kind of system in 1952 as a model for biological morphogenesis [6]. The general form of these equations is

$$\frac{d\vec{I}}{dt} = \vec{F}(\vec{I}) + D\nabla^2\vec{I} \quad (6)$$

for the evolution of the "field" $\vec{I} = (I_1, I_2, \ldots, I_k)$ of concentration variables. The function $\vec{F} = (F_1, F_2, \ldots, F_k)$ represents the local "reaction" dynamics of these variables without diffusion. D is a matrix describing the spatial coupling and diffusion rate of the concentration variables. For linear $\vec{F}$, Turing showed that this system gives rise to spatial patterns that can oscillate temporally. He also considered the addition of a noise term and its effect on the selection of spatial patterns.

These equations naturally take into account spatial diffusion with the Laplacian operator on the RHS of eq. (6). Furthermore, the continuous time derivative and the local reaction dynamics can be used to implement a temporal low pass filter. Thus. reaction–diffusion models can be constructed that satisfy the basic criteria already laid down for video feedback. Video feedback differs from Turing's reaction–diffusion models because of a nonlocal spatial coupling resulting from the spatial rotation and magnification. In direct analogy with the previous arguments, the proposed reaction–diffusion equation for color video feedback dynamics is

$$\frac{d\vec{I}(\vec{x})}{dt} = \underline{L}\vec{I}(\vec{x}) + sf\vec{I}(bR\vec{x}) + \sigma\nabla^2\vec{I}(\vec{x}), \quad (7)$$

where the parameters s, f, b, $\underline{L}$, and R, are as before, and σ is a matrix summarizing the spatial diffusion rate. The first term on the RHS of eq. (7) is the "old image", the next term is the nonlocal "incoming image", and the last is the diffusion coupling. For spatial structure and temporal behavior well below the spatial and temporal frequency cutoffs discussed above, this model should be valid. As will be seen in the next section, video feedback dynamics has very similar phenomenology to that of chemical and biological systems described by this type of model. The reaction–diffusion model provides a conceptual simplicity as well as simpler notation. In fact, video feedback can be used to experimentally study this widely used class of models for spatio-temporal complexity.

The previous iterated functional equation model eq. (4) can be derived from eq. (7) upon discretization. Eq. (7) is the differential form of eq. (4), an integro-functional difference equation. A digital computer simulation of this continuum model naturally involves spatial and temporal discretization. Thus, as far as verifying the models by digital simulation, it is a moot point as to which is better, the iterated functional equation or reaction–diffusion model.

Having constructed these models, the burning question is whether their dynamics describe that actually found in real video feedback systems. For the very simplest behavior there is hope that the equations can be solved analytically. In general, though, simulating the models in a more controlled environment on a digital computer, for example, seems to be the only recourse [7]. After describing the dynamics typically observed in a real video feedback system in the next section, I will come back to the results of just such a digital simulation.

4. Video software

The models and discussion of video physics in the last section may have given an impression of simplicity and straightforwardness in understanding video feedback dynamics. The intent in this section is to balance this with a little bit of the richness found in an actual color video system. An overview of the observed dynamics will be presented initially from a dynamical systems viewpoint. I will also address the appropriateness of this framework for some of the more complex dynamics. Then a brief description of a movie on video feedback follows. Stills from the movie illustrate some of the curious features of video feedback dynamics. And finally, these "experimental" results will be compared to those from preliminary digital computer simulations.

Video feedback dynamics can be roughly categorized as in table II. For the simplest temporal behavior, descriptive terms from dynamical systems seem appropriate as in the first four behavior types. At first, let's ignore any possible spatial structure in the images. When a stable time-independent image is observed, it corresponds to a fixed point in the image space $\mathscr{F}$. Much of the behavior seen for wide ranges of control parameters falls into this category.

Thus on the large scale video systems are very stable, as they should be in order to operate properly in a wide range of environments. For extreme parameter settings, such as small rotation, low contrast, large demagnification, and so on, equilibrium images are typically observed. For example, when the zoom is much less than unity then one observes an infinite regression of successively smaller images of the monitor within the monitor within .... The image is similar to that

Table II
Video feedback dynamics

Observed	Attractor in image space
equilibrium image	fixed point
temporally repeating images	limit cycle
temporally aperiodic images	chaotic attractor
random relaxation oscillation	limit cycle with noise-modulated stability
spatially decorrelated dynamics (e.g. dislocations)	quasi-attractor with local temporal dynamics: fixed point; limit cycle; chaotic attractor
spatially complex image	spatial attractor: fixed point; limit cycle; chaotic attractor (?)
spatially and temporally aperiodic	nontrivial combination of the above

seen when two mirrors face each other. With a bit of rotation the infinitely regressing image takes on an overall "logarithmic spiral" shape that winds into the origin.

When the parameters are set to moderate values, one of the first non-trivial dynamics to appear is a simple oscillation. This would be a limit cycle in image space: a sequence of dissimilar images that after some time repeats. Because entire images repeat, individual points on the screen exhibit periodic behavior. Consequently, the values of intensity at a point cycle repetitively.

At parameter values nearby often lie temporally aperiodic image sequences. Chaotic attractors in image space are most likely a good description of this behavior type in the simplest cases*. When non-repeating images are reached from limit cycles with the change of a parameter, the bifurcation occurs in one of (at least) three ways:

1) Simple lengthening of the limit cycle period, until it is sufficiently long to be effectively aperiodic: for example, going from a limit cycle of 10 seconds to one of hours. New images are introduced, but are not sufficiently similar to be considered as close "recurrences".

2) The introduction of subharmonics at frequencies lower than that of the original limit cycle: these subharmonics are small modulations of the image's geometric structure. The overall image sequence remains the same, but differs in the modulated detail.

3) Suddenly at some critical parameter value, the limit cycle disappears and aperiodicity set in.

A very telling indication that complex behavior lies at nearby parameter settings comes from slightly perturbing the system. This can be done most conveniently by waving a finger between the monitor and camera. Once perturbed, the nearby complexity reveals itself by long and convoluted transients as the system settles down to its original simple fixed point or limit cycle. The closer in parameters to aperiodic behavior, the longer the transients. The simple dynamics discussed so far are globally stable in just this sense of returning to the same image(s) when perturbed. Of course, one can perturb the system too much, knocking it into another basin of attraction and so losing the original behavior. It is a common experience, in fact, that hand-waving perturbations will leave the screen dark, with the system requiring a "positive" stimulus of light from some source to get back to its initial attractor.

At large zoom, or spatial magnification, the system noise is readily (and exponentially) amplified. This regime is dominated by bursts of light and color. Depending on the controls, the bursts can come at regular intervals or at random times. Also, the particular features of the bursts, such as color, intensity, or even the pattern, can be the same or aparently randomly selected. This behavior is quite reminiscent of a limit cycle with (noise) modulated stability [9].

The dynamics discussed so far is simple in the sense that its temporal features are the dominant aspect. No reference was made to spatial structure as the temporal dynamics was readily distinguished from it. A more precise way to make this distinction is in terms of whether the behavior at a suitably chosen point captures the dynamics [8]. Using intensity data from this point, if a simple attractor can be reconstructed, then the behavior is of a simple type that can be *decomposed* into temporal and spatial components. The last entries in table II are an attempt to indicate that there is much more than this simple decomposable dynamics. Indeed, the spatial structure and its interaction with the temporal dynamics are what makes video feedback different from other systems with complex dynamics, like chaotic nonlinear oscillators. But this difference presents various (intriguing) difficulties, especially because a dynamical system description does not exist for spatial complexity [10]. Nonetheless, a qualitative description is possible and, hopefully, will lead to the proper theoretical understanding of spatial dynamics.

* In this case, given a time series of intensity values at a point, it is possible to "reconstruct" a state space picture of the attractor [8].

Much of the following description, and the categorization used in table II, is based on *observed* similarities in spatial structure. While it may be very difficult to unambiguously state what a complex image is, we as human beings can easily discern between two images and can even say some are "closer" than others in structure. I am not currently aware, however, of any mathematical definition of "closeness" for spatial structure that is of help with the dynamics observed in video feedback. Such a concept would be of immense value in sorting out complex dynamics not only in video feedback but in many other branches of science.

To denote images that are observed to be similar, but different in spatial detail, I introduce the phrase "quasi-attractor" for the associated object in state space. These state space objects appear to be globally stable to small perturbations and it is in this sense that they are attractors. Once perturbed, the video system returns to similar images, although in spatial detail they may be slightly altered from the original.

A good example of quasi-attractors is the class of images displaying *dislocations*. This terminology is borrowed from fluid dynamics, where dislocations refer to the broken structure of convective rolls in an otherwise simple array. Dislocations are regions of broken symmetry where the flow field has a singularity. The formation of this singularity typically requires a small, but significant, energy expenditure*. In video feedback, dislocations appear as inter-digitated light and dark stripes. The overall pattern can be composed of regular parallel arrays of alternating light and dark stripes with no dislocations, and convoluted, maze-like regions where stripes break up into shorter segments with many dislocations. The boundaries between segment ends form the dislocations. They can move regularly or wander erratically. Dislocations form in pairs when a stripe breaks in two. They also annihilate by coalescing two stripes. Dislocations make for very complex, detailed patterns whose temporal evolution is difficult to describe in terms of dynamical systems because of their irregular creation and annihilation. Nonetheless, when perturbed very similar images reappear. A quasi-attractor would be associated with global features, such as the relative areas of regular stripe arrays and dislocation regions, the time-averaged number of dislocations, or the pattern's gross symmetry.

Dislocations fall into the behavior class of *spatially decorrelated dynamics*. Moving away from one point on the screen, the spatial correlations decay rapidly enough so that eventually there is no phase relationship between the behavior of different regions. The governing dynamics in any one area is similar to that of other areas. The local behavior, however, can take on the character of a fixed point, limit cycle, or chaotic attractor. Thus while globally stable, the entire image cannot be described by a *single* attractor in the conventional sense of dynamical systems theory. This behavior type has been studied quantitatively in simple nonlinear lattice models [13]. Spatially decorrelated dynamics apparently is the cause of heart fibrilation that results in sudden cardiac death [14].

The existence of *spatial attractors* that describe an image is another useful notion in classifying video dynamics. Intensity values as a function of a "pseudo-time" can be obtained by following along a simple parametrized curve on the screen. These values then can be used to reconstruct a "state space" picture [8] that captures some features of an image's structure. These features naturally depend on the type of curve selected. For example, data from a circle of fixed radius elucidates the rotational symmetry in an image. Similarly, data from along a radial line allows one to study radial wave propagation caused by magnification. The reconstruction of spatial attractors has been carried out for the above-mentioned lattice models [13].

* Both Couette flow [11] and Bénard convection [12] exhibit this phenomenon. In nematic liquid crystal flow these are called *disclinations*. Similar structures appear in spin systems, such as magnetic bubble devices, and in the formation of crystals. Turing's discussion [6] of "dappled patterns" in a two-dimensional morphogen system is also relevant here.

The rough classification is not yet complete. There are also image sequences that appear to be combinations of spatially-decorrelated dynamics and complex spatial attractors. The latter entries in table II indicate these possibilities.

The interaction of spatial and temporal dynamics makes it very difficult to describe the more complex behavior in any concise manner. To alleviate this problem a short video tape was prepared to illustrate the types of behavior in table II [4]. The movie is particularly effective in giving a sense of the temporal evolution, stability, and richness of video feedback dynamics. An appreciation of the spatial complexity can be gleaned in a few stills from the movie. (See plates 1–7.) This will compensate hopefully those readers who do not have access to a video feedback system or who have not seen the movie.

The examples have a few common features. Regarding parameter settings, they were all made at rotations of approximately 40 degrees and with spatial magnifications slightly less than unity, unless otherwise noted. The discreteness caused by the finite resolution is apparent in each figure. Note that the spatial structures are typically many pixels in extent, so that the discreteness does not play a dominant role.

Plate 1 presents a typical nontrivial equilibrium image, or fixed point. It has an approximate nine-fold symmetry that comes from the rotation angle: $360/40 = 9$. The intensity at each point as a function of angle is periodic, with periods not greater than nine. The overall spatial symmetry as a function of rotation ϕ exhibits a "symmetry locking" highly reminiscent of that found in temporal frequency locking in nonlinear oscillators [3]. One noteworthy similarity is that the parameter window for which a given symmetry dominates decreases in width with increased order of the symmetry. For example, spatially symmetric images of period 31 occur for a much smaller rotation range those with period 9 symmetry.

One image out of a long limit cycle is shown in plate 2. The limit cycle period was approximately 7 seconds. Initially, a green disk nucleates at the center of a homogeneous light blue disk. The green disk grows to fill 80% of the illuminated area leaving a blue annulus. A red disk then nucleates inside the green disk, along with an outside ring of nine dots. The oscillation consists largely of the radially outward moving red disk, that intercepts the inward propagating dots. The still is taken at the moment of collision. The disk expands engulfing the dots and the green annulus, then itself is over taken by the inside boundary of the blue annulus that moves inward. The outer boundary of the red disk then recedes before the blue annulus. The screen then eventually becomes entirely light blue, at which moment the center nucleates a growing green disk, and the cycle repeats. This limit cycle was stabilized by a very small marking near the screen's center*.

Plate 3 shows a still from a sequence of images with slowly moving dislocations. Toward the outside there is a "laminar" region of stripes. Moving inward from this, the first ring of nine dislocations is encountered. These were seen to move smoothly counter-clockwise. The center, however, periodically ejected thin white annuli that propagated out radially, only slowly acquiring clockwise rotation. The interface between the inner and outer regions caused the intervening maze-like dislocation pattern. The entire image shows a high degree of nine-fold symmetry although in the dislocation region it is quite complex.

Spiral patterns are quite abundant, as one expects from a transformation with rotation and magnification. Plate 4 illustrates a *logarithmic spiral* that dynamically circulates clockwise outward. Temporally, the behavior is periodic with color and structure flowing outward from the center. The rotation here is $\phi = -30$ degrees. The logarithmic spiral can be easily described as a parametrized curve with angle ϕ and scaling b controls as follows

$$(x, y) = (bt \cos(\phi \log t), bt \sin(\phi \log t)),$$

* One evening this cycle was allowed to oscillate for two hours with no apparent deviation from periodicity before the power was turned off.

with $t \in [0, 1]$. Such structure and periodic coloring occur often in organisms, such as budding ferns and conch shells.

With relatively high zoom, or large spatial magnification greater than unity, noise in intensity and spatial structure is exponentially amplified. A common manifestation of this is periodic or random bursts. Plate 5 shows a snapshot of a developed burst that had spiralled counterclockwise out of the center in about one second. After a burst the screen goes dark with faint flickering, until another fluctuation occurs of sufficient magnitude to be amplified into a spiralling burst. The video system's finite resolution can be seen as a graininess on a scale larger than the intrinsic discreteness.

Luminance inversion stabilizes images by amplifying contrast. Black regions map into white and colors map to their opposite. This sharpens boundaries between dark, light, and colored areas in an image. Section VI of ref. 2 discusses this stabilizing effect in more detail. Plate 6 shows an example of the "pinwheels" that dominate the images found with luminance inversion*. The rotation for this photo was $\phi = -90$ degrees. By adjusting the rotation, focus, and/or hue, controls the pinwheels are seen to move either clockwise or counterclockwise. Winfree discusses similar "rotating waves" of electrical impulses that cause the heart's coordinated beating. Plate 6 should be compared to the figure on page 145 of ref. 14.

Plate 7, also made with luminance inversion, is a snapshot of outward spiralling "color waves". These are very reminiscent of the ion concentration waves found in the Belousov–Zhabotinsky chemical reaction [15]. The rotation parameter here is roughly $\phi = -40$ degrees. As in the above pinwheels, every point in the image has a well-defined temporal phase, except for the center where there is a phase singularity.

A digital simulation based on eqs. (4) and (7) captures some of the gross features of video feedback. To this extent the proposed models are correct. It is still an open question as to whether they reproduce the detailed spatio-temporal dynamics. Such comparison is a difficult proposition even in modeling temporal chaos alone. Digital simulations are many orders of magnitude slower than the space–time analog simulations of video feedback. And for this reason it is difficult, given model equations, to verify in detail and at numerous parameter settings their validity. To date digital simulations [7] have reproduced the following features typical of video feedback:

1) equilibrium images with spatial symmetry analogous to Turing's waves [6];

2) fixed point images stable under perturbation;

3) meta-stability of fixed point images: sufficiently large perturbations destroy the image;

4) logarithmic spirals;

5) logarithmic divergence when the rasters are not centered.

At this preliminary stage of digital simulation it is not possible to discuss much in detail. In fact, it may be a long time until extensive digital simulations are carried out on the proposed models. The construction of, or use of pre-existing, special purpose digital image processors to simulate video feedback may be more feasible than using conventional digital computers. The next and final section comes back to address these questions of future prospects for understanding video feedback.

5. Variations on a light theme

Video feedback is a fast and inexpensive way to perform a certain class of space–time simulations. It also provides an experimental system with very rich dynamics that is describable in some regimes by dynamical systems theory, while in other regimes it poses interesting questions about extending our current descriptive language to spatial complexity.

One goal in studying video feedback is to see whether it could be used as a simulator for dynamics in other fields. Turing's original proposal of reaction-diffusion equations for biological mor-

* Bob Lansdon introduced me to these pinwheel images. See also ref. 2.

phogenesis comes to mind, as well as the image processing [16] and hallucinogenic dynamics [17] of the visual cortex. Naturally, the first task in this is to understand video feedback itself as completely as possible. Toward this immediate end, I have proposed models based on video physics and presented an overview of the possible behavior in a particular color video system. The next steps in this program are to make a more quantitive study of the attractors and bifurcations with calibrated video components. Data from these experiments would be analyzed using techniques from dynamical systems to (i) reconstruct state space pictures of the simpler attractors, and (ii) quantify the unpredictability of the simple aperiodic behavior.

A second approach to understanding video feedback dynamics is to study other configurations of video components. The possibilities include:

1) masking portions of the screen to study the effect of boundary conditions;

2) optical processing with filters, lenses, mirrors, and the like;

3) using magnets to modulate the monitor electron beam scanning;

4) connecting two camera–monitor pairs serially, thus giving twice as many controls;

5) nonlinear electronic processing of the video signal;

6) inserting a digital computer into the feedback loop via a video frame buffer.

The possible modifications are endless. But, hopefully, they will help point to further understanding and lead to applications in other fields.

Variations (5) and (6) may lead to the most fruitful applications of video feedback. For example, they allow one to alter the governing rules in simulations of two-dimensional local and nonlocal automata. In this process an image is stored each raster time. Each pixel and its neighbors are operated on by some (nonlinear) function. For rapid ("real-time") simulation this function is stored in a "look-up" table. The pixel value and those of its neighbors form the input to the table. The table's result then becomes the pixel's new value that is stored and displayed. This is a very general configuration. With video feedback one has simple control over the nonlocality of the rules using rotation and spatial magnification, and over the number of neighboring pixels using the focus.

A monochrome system, employing an intensity threshold to give crisp black and white images, could be used to simulate binary cellular automata. This restriction on the intensity range falls far short of the possible pixel information in video systems. Indeed, as discussed in the appendix, color systems are capable of transmitting roughly 20 bits of information per pixel. This includes a random "noise floor" for small signals. Generalizing cellular automata, from a few states per site to many, leads to lattice dynamical systems [13]. This corresponds in the video system to removing the above thresholding. Thus this video configuration will be especially useful in the experimental study of lattice dynamical systems and in the verification of analytic and numerical results, such as spatial period-doubling, found in some nonlinear lattices [13].

A number of video image processors are available, both analog and digital. Many have been constructed solely according to their aesthetic value by video artists. Certainly, among this group there is a tremendous amount of qualitative understanding of video dynamics. At the other extreme of the technical spectrum, some of the emerging supercomputers have adopted architectures very similar to that of video feedback systems. These machines would be most useful in detailed quantitative simulations. And, in turn, video feedback might provide an inexpensive avenue for initial study of simulations planned for these large machines.

Physics has begun only recently to address complex dynamical behavior. Looking back over its intellectual history, the very great progress in understanding the natural world, with the simple notions of equilibrium and utter randomness, is astounding. For the world about us is replete with complexity arising from its intimate interconnectedness. This takes two forms. The first is the recycling of information from one moment to the next, a temporal inter-connectedness. This is feedback. The second is the coupling at a given time between different physical variables. In globally stable systems, this often gives rise to nonlinearities. This inter-connectedness lends structure to the chaos of microscopic physical reality that completely transcends descriptions based on our traditional appreciation of dynamical behavior.

From a slightly abstract viewpoint, closer to my personal predelictions, video feedback provides a creative stimulus of behavior that apparently goes beyond the current conceptual framework of dynamical systems. Video feedback poses significant questions, and perhaps will facilitate their answer. I believe that an appreciation of video feedback is an intermediary step, prerequisite for our comprehending the complex dynamics of life.

Acknowledgements

I am particularly indebted to Ralph Abraham for introducing me to video feedback a number years ago. Special thanks are due to Doyne Farmer and the Center for Nonlinear Studies, Los Alamos National Laboratory, for the support and encouragement of this project. Larry Cuba generously loaned his video equipment for Plates 6 and 7. Elaine Ruhe was especially helpful in the preparation of the video tape and stills. I would also like to thank the Automata Workshop participants who played with the video feedback demonstration and discussed their ideas with me. Particular thanks go to Bob Lansdon, Alice Roos, Otto Rössler, and Art Winfree, for useful discussions on video feedback.

Appendix A

Video physics

There are many types of camera pickup tubes, but for concreteness I will concentrate on the common vidicon tube and describe how it converts an image to an electronic signal. The vidicon relies on the photoconductive properties of certain semiconductors (such as selenium). When light is incident on these materials their electrical resistance is reduced. Photoconductors can have quite large quantum efficiencies, approaching 100%, with virtually all the incident photon energy being converted to mobilizing electrons in the material. Once energized these electrons diffuse in an ambient electric field.

The vidicon takes advantage of these mobile electrons in the following way. (Refer to fig. 3.) An image is focused on a thin *photoconducting layer* (A) approximately one square inch in size. Spatial variation in an image's light intensity sets up a spatial distribution of mobile electrons. Under influence of a small bias field these diffuse toward and are collected at the transparent *video signal pickup* conductor (B). During operation the photoconductor/pickup sandwich acts as a leaky capacitor with spatially varying leakage: the more incident light, the larger the local leakage current. The *electron beam* (C) from the vidicon's cathode scans the back side of the photoconductor depositing electrons, restoring the charge that has leaked away, and hence, bringing it to a potential commensurate with the cathode. The coils (D) supply the scanning field that moves the electron beam over the photoconductor. They are driven synchronously with the horizontal and vertical raster timing circuits (top of diagram). The output video signal corresponds to the amount of charge locally deposited by the beam at a given position during its scan. This charge causes a change in the leakage current and this change is picked up capacitively and then amplified.

The important features of this conversion process, aside from the raster scanning geometry already described, are

1) the diffusion of electrons as they traverse the photoconductor; and

2) the local storage and integration of charge associated with the light incident during each raster time.

The diffusion process directly limits the attainable spatial resolution. This places an upper bound on the number of horizontal lines and the number of *pixels* (distinct picture elements) within each line. The effect on spatial patterns is that there can be no structure smaller than this diffusion limit. Another interpretation of this is that, over the period of several rasters, there is a diffusive coupling between elements of an image.

The high spatial frequency cutoff can be easily estimated. The electron beam forms a dot on the photoconductor's backside approximately 1 to 2 mils in diameter. Diffusion then spreads this out to roughly twice this size by the time these electrons have traversed the layer, yielding an effective 3 to 4 mils minimum resolution. For a vidicon with a one inch square photoconducting target, this results in a limit of 250 to 300 pixels horizontally and the same number of lines vertically. These are in fact nominal specifications for consumer quality cameras. Additionally, although the raster geometry breaks the image into horizontal lines, the resolution within each line is very close to that given by the number of scan lines. It will be a reasonable approximation, therefore, to assume that the spatial frequency cutoff is isotropic.

In a similar manner the charge storage and integration during each raster time places an upper limit on the temporal frequency response of the system. In fact, this storage time τ_s can be quite a bit longer than the raster time τ_r of 1/30 second. A rough approximation to this would be $\tau_s \approx 10\tau_r \approx 1/3$ second. Thus the system's frequency response should always be slower than 3 Hz. And this is what is observed experimentally. Even the simplest (linear) model for video feedback must contain spatial and temporal low pass filters corresponding to the above limitations.

The optical system that forms the image on the photoconductor has spatial and temporal bandwidths many orders of magnitude greater than the vidicon itself. Hence these intrinsic optical limitations can be neglected. The optical system controls, however, are quite significant. The focus, for example, can affect an easily manipulated spatial diffusion by moving the image focal plane before or behind the photoconductor. In addition, by adjusting it to one side of exact focus the diffusion orientation can be inverted. Very small changes in the zoom, or spatial magnification, can have quite large qualitative effects because the image information repetitively circulates in the feedback loop. A spatial magnification greater than unity increases exponentially with the number of passes through the loop. Similarly, adjusting the admitted light with the f/stop can cause the light in an image to dissipate completely when set below some intrinsic threshold.

The image intensity can again be adjusted with the brightness control on the monitor, perhaps to compensate for the camera's f/stop setting. The brightness adjusts the DC intensity level of the video signal, while the contrast amplifies its dynamic range, or the AC portion of the video signal. High contrast will amplify any noise or spurious signal into an observable flickering of the image. A monochrome monitor's screen (E) is coated with a uniform layer of phosphor that emits light when struck by the electron beam (G). Using the monitor's driving coils (D), the raster synchronizing circuits move the beam to the appropriate position on the screen for the incoming video signal. This signal modulates the beam's intensity (F). The screen's spatial resolution is effectively continuous with a lower bound significantly less than that imposed by the vidicon resolution and by the finite number of scan lines. Additionally, the phosphor stores each raster for a short time to reduce flickering. Thus there is another image storage element in the feedback loop. The phosphor's *persistence* is typically a single raster time and so it can be neglected compared to the vidicon's storage time.

There are a number of sources of error, or deviations from the idealized video feedback system. Here I will briefly mention a few that could be taken, more or less easily, into account in the modeling, but for simplicities sake will not be included. The first omission that I have made in describing the functioning of video systems, is that the bulk of them transmit two *interlaced* half-rasters, or *fields*, every sixtieth of a second. A complete raster is still formed every thirtieth of a second, but the successive images appear to flicker less than without interlaced fields. Since the time scale of this is much less than the image storage and integration time of the vidicon it can be neglected.

A second and important error source is the intrinsic noise of the intensity signal. A number of physical processes contribute to this noise. The discreteness of the quantum processes and the electron charge produce resistive noise in the photoconductor. The electronic amplifiers for the signal also introduce noise. The net effect though is a signal to noise ratio of about 40 db. This translates into about 10 mV white noise superimposed on the 1 V standard video signal, or into about 1% fluctuation in the intensity of pixels on the monitor's screen.

The photoconductor's monotonic, but nonlinear, current output i_0 as a function of light intensity I_i adds a third error. For vidicons $i_0 \sim I_i^\gamma$, with $\gamma \in [0.6, 0.9]$. Furthermore, this response function saturates above some intensity threshold I_{sat}. Vidicon photoconductors also exhibit a nonuniform sensitivity of about 1% over the target region.

When the camera is very close to the monitor, there is significant geometric distortion due to the screen's curvature. Geometric distortion also arises from other errors in the system, such as the adjustment of the horizontal and vertical raster scanning circuitry. These distortions can be reduced to within a few percent over the image area. Finally, within the monitor there are saturating nonlinearities in its response to large intensity signals and high brightness or high contrast settings. This list is by no means exhaustive, but at least it does give a sense of the types of errors and their relative importance.

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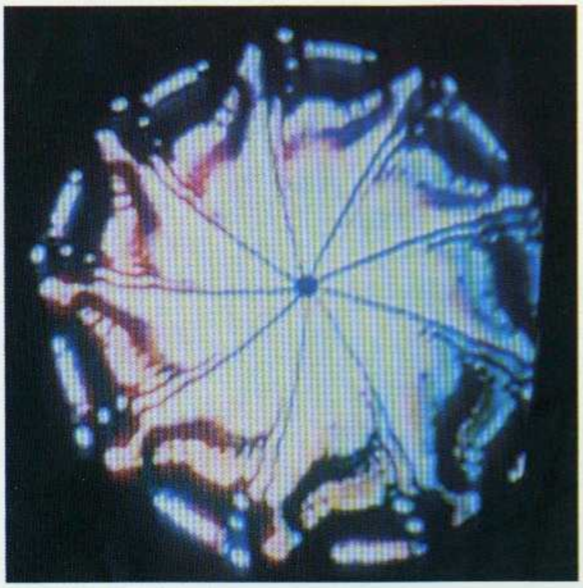

Plate 1. Equilibrium image.

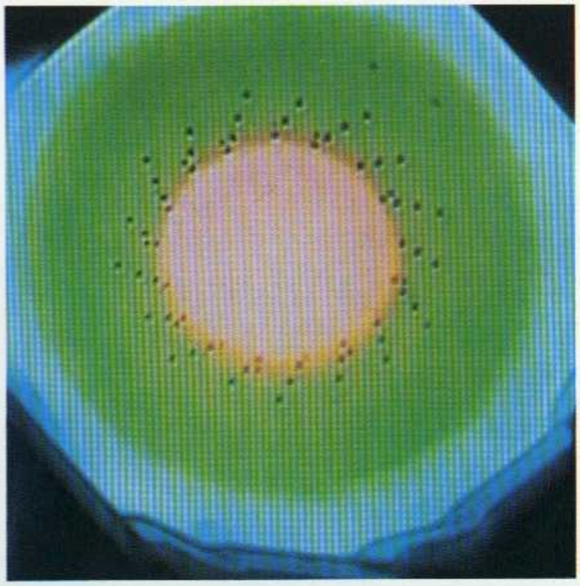

Plate 2. Snapshot of a limit cycle.

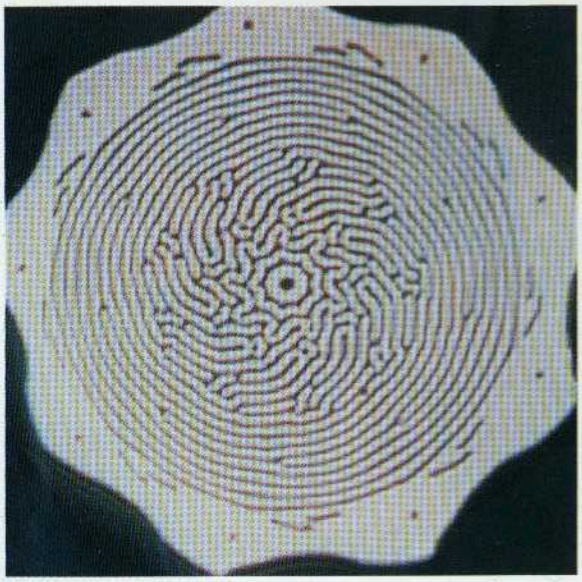

Plate 3. Dislocations.

Plate 4. Logarithmic spiral.

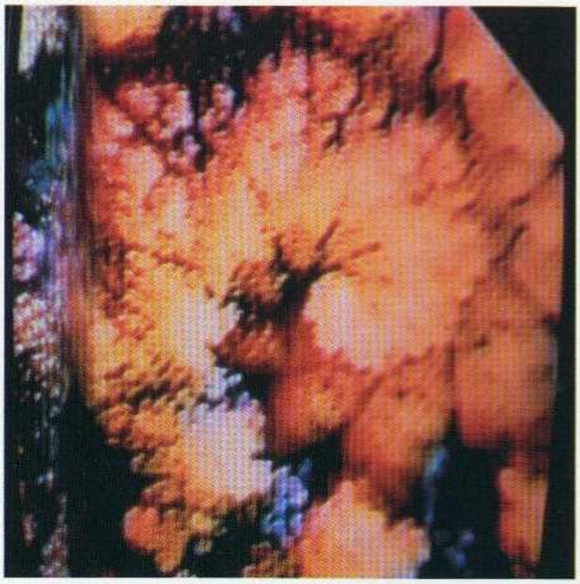

Plate 5. Relaxation oscillation at high zoom.

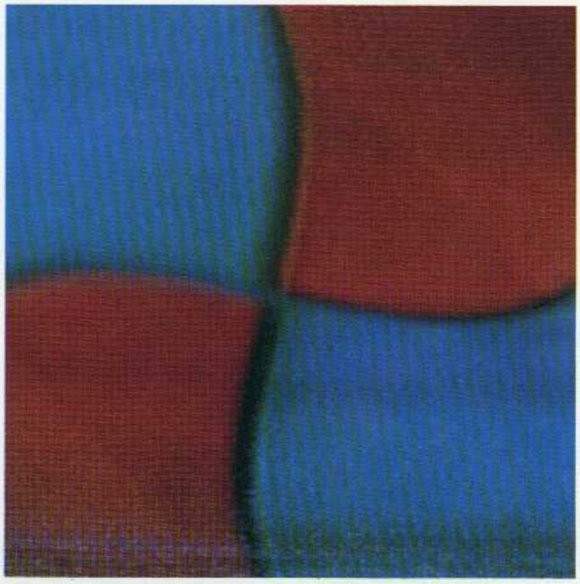

Plate 6. Pinwheels.

Plate 7. Spiral waves.